A Decomposition Method With Minimal Communication Volume for Parallelization of Multi-dimensional FFTs

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Motivation

Fast Fourier Transform (FFT)
An essential kernel in science and engineering.

Parallelization of FFT
- Existing domain decomposition methods pre-define the dimensions of decomposition and therefore are not adaptive.
- The order of data transpose may have an impact on the volume of communication.
- Little work has explored beyond 3-D FFTs, while 4-D and 5-D FFTs also have various applications.

Purpose

Develop a domain decomposition method with minimal volume of communication for the parallelization of multi-dimensional FFTs.

Objectives
- Minimal volume of communication.
- Applicable to 3-D, 4-D, 5-D FFTs, and beyond.
- Adaptively decompose in the lowest dimensions depending on the number of processes.
- Follow the most communication-efficient order.
- Able to work with an arbitrary number of processes.

Method

1. Adaptive decomposition
- Translate the multi-dimensional data into one-dimensional data, and divide the resultant one-dimensional data equally to the processes using a block distribution.
- Treat the dimensions in a specific order: abc, cba, bca, etc.
- Decompose in the lowest possible dimensions depending on the number of processes.

2. Transpose-order awareness
- The adaptive decomposition provides plenty of transpose orders.
- Different order results in different volume of communication.
- Choosing a proper order reduces the volume of communication.

Evaluation

3. Transpose order and volume of communication
- \((M-1)!\) \(M\) transpose orders for \(M\)-dimensional FFTs.
- 8, 1296, and 7962624 transpose orders for 3-D, 4-D, and 5-D FFTs, respectively.
- Analyses are computationally performed.

Summary

Our method
- Adaptive decomposition + Transpose-order awareness.
- Decompose in the lowest dimensions, and follow the most communication-efficient transpose orders.
- Numerical results show good performance and scaling property.

Future work
- Improve memory usage and communication implementation.
- Extend our implementation to \(M\)-D FFTs.

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References