# THE FAST ICA ALGORITHM
## (OVERVIEW)

R. Frank K. Glass
Neuroinformatics Center

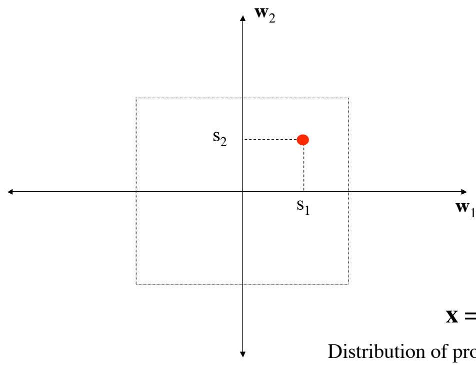
## What is ICA?

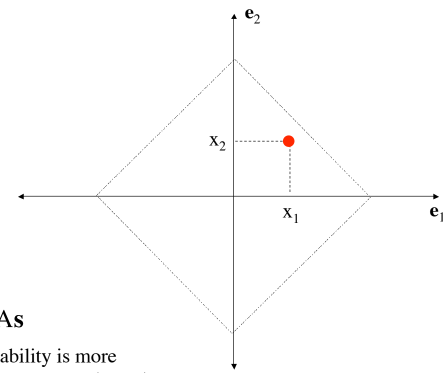
ICA separates n statistically independent inputs, with non-Gaussian distributions, which have been linearly mixed into n output channels.

## Statement of the Problem

- $\mathbf{s}$ = the random vector of statistically independent components $s_1, \ldots, s_n$.
- $\mathbf{x}$ = the random vector whose elements are the mixtures $x_1, \ldots, x_n$.
- A is the unknown 'mixing' matrix.
- $x_i = a_{i1}s_1 + \ldots + a_{in}s_n$, for i = 1…n,
  or
  $\mathbf{x} = A\mathbf{s}$.
- ICA 'finds' the inverse of A, $W = A^{-1}$, such that
  $\mathbf{s} = W\mathbf{x}$.

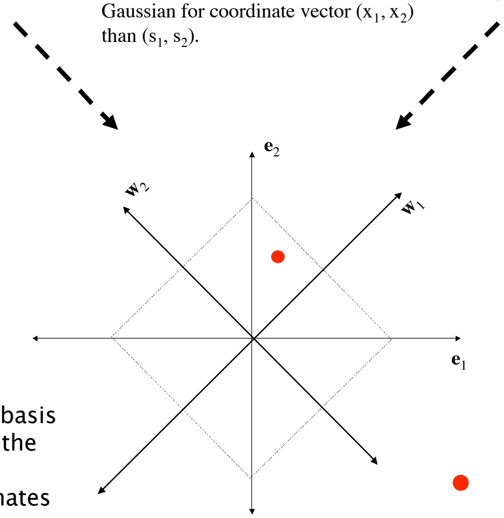
## Theoretical Basis, Heuristically Explained…




$\mathbf{x} = A\mathbf{s}$

Distribution of probability is more Gaussian for coordinate vector $(x_1, x_2)$ than $(s_1, s_2)$.




$\mathbf{x} = A\mathbf{s}$ can be considered a change of basis from $\{\mathbf{w}_1, \mathbf{w}_2\}$ to $\{\mathbf{e}_1, \mathbf{e}_2\}$. ICA recovers the basis $\{\mathbf{w}_1, \mathbf{w}_2\}$ in which the probability distribution of the data, whose coordinates are the vector $(s_1, s_2)$, is maximally non-Gaussian.

$$\bullet = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{e}} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}_{\mathbf{w}}$$

## Conclusion

A result of the Central Limit Theorem is that the probability distribution of a linear combination of statistically independent random variables is more 'Gaussian' than any of the random variables which comprise the sum.

Since $x_i = a_{i1}s_1 + \ldots + a_{in}s_n$, $\mathrm{pdf}(x_i)$ is more Gaussian than $\mathrm{pdf}(s_i)$, i = 1…n.

If each row, $\mathbf{w}_i^T$, of the unmixing matrix W is computed to minimize the non-Gaussianity of $\mathbf{w}_i^T\mathbf{x}$, for i = 1…n, then we will have extracted the independent components. These components constitute the $\{\mathbf{w}\}$ basis.

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = \mathbf{s} = W\mathbf{x} = \begin{pmatrix} \mathbf{w}_1^T\mathbf{x} \\ \vdots \\ \mathbf{w}_n^T\mathbf{x} \end{pmatrix}$$