FINITE ELEMENTS AND FINITE DIFFERENCE HUMAN HEAD MODELING: FORWARD PROBLEM

S. Turovets[1] K. Glass[1] A. Malony[1] V. Volkov[2]

[1]Neuroinformatics Center [2]Institute of Mathematics

INTRODUCTION:PHYSICS OF EEG/MEG

Fundamental problems in electroencephalography (EEG) and magnetoencephalograpy (MEG), in particular , source localization and impedance imaging require modeling and simulating the associated bioelectric fields. The relevant frequency spectrum in EEG and MEG is typically below 1 kHz, and most studies deal with frequencies between 0.1 and 100 Hz, therefore, the physics of EEG/MEG can be well described by the quasi-static approximation of Maxwell's equations.

The forward problem can be stated as follows: given the positions, orientations and magnitudes of dipole current sources, as well as geometry and electrical conductivity of the head volume , Ω, calculate the distribution of the electrical potential on the surface of the head (scalp), Γ_Ω. Mathematically, it means solving the linear Poisson equation[1]:

$$\nabla \bullet (\sigma \nabla \phi) = \nabla \bullet \mathbf{J}^s, \quad \text{in } \Omega \qquad (1)$$

with no-flux Neumann boundary conditions on the scalp:

$$\sigma(\nabla \phi) \bullet \mathbf{n} = 0, \text{ on } \Gamma_\Omega . \qquad (2)$$

Here $\sigma = \sigma_{ij}(x,y,z)$ is an inhomogeneous tensor of the head tissues conductivity. Having computed potentials ϕ and current densities $\mathbf{J} = -\sigma(\nabla\phi)$, the magnetic field B can be found through the Biot-Savart law.

The inverse problem implies fitting the computed and measured data to extract information on location of the sources or the internal head tissues properties and usually involves the large number of runs for the forward problem. This is why the computational methods for the forward problem which are stable, fast and eligible for parallelization are of paramount importance.

COMPUTATIONAL METHODS

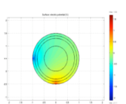
The main idea behind the FE or FD methods is to reduce a continuous problem with infinitely many unknown field values to a finite number of unknowns by discretizing the solution region into elements.

The FD method is generally the easiest method to code and implement, but it usually requires special modifications to define irregular boundaries, abrupt changes in material properties, and complex boundary conditions. While typically more difficult to implement, the FE/BE methods are usually preferred for problems with irregular, inhomogeneous domains and mixed boundary conditions.

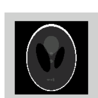
Application of each of the previous approximation methods to (1) yields a system of linear equations of the form $A\phi = b$, which must be solved to obtain the final solution. The solution techniques can be broadly categorized as ***direct*** and ***iterative*** solvers[2] (cf. Diagram). The choice of the particular solution method is highly dependent upon the approximation technique employed to obtain the linear system, upon the size of the resulting system, and upon accessible computational resources.

In the present study we are focusing on investigation of the capabilities of the commercial package FEMLAB as an input FE mesh generator and a solver for bioelectric field problems in simplified phantoms and realistic geometries, as well as the FD alternative direction implicit algorithms [3,4].

HEAD PHANTOMS: FEMLAB



4-shell sphere model
- 2 points current injection

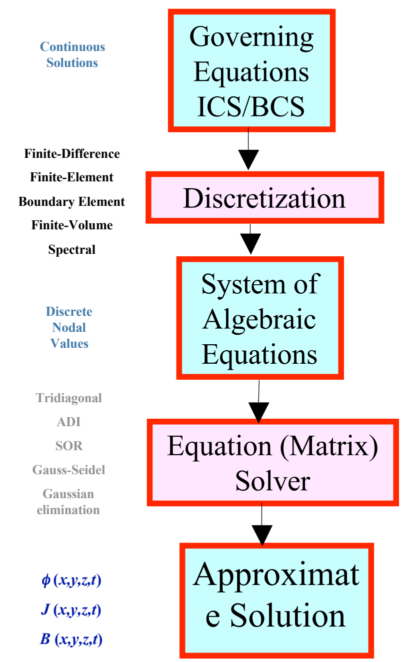


Ellipsoid model

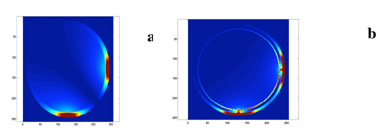
Tissues parameters in 4-shell model

Tissue type	$\sigma(\Omega^{-1}m^{-1})$	Reference	Radius (cm)
Brain	0.25	Geddes(1967)	8
CSF	1.79	Baumann(1997)	8.2
Skull	0.018	Law (1993)	8.7
Scalp	0.44	Burger(1943)	9.2

COMPUTATIONAL PROCEDURE

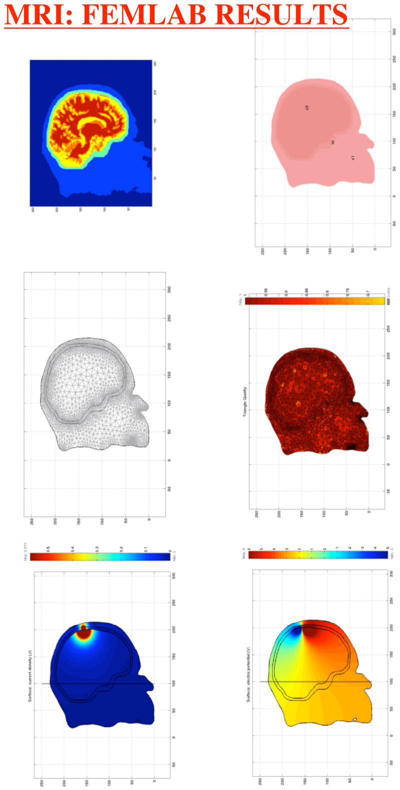


FD ADI Algorithm: 3D - 4-shell sphere model

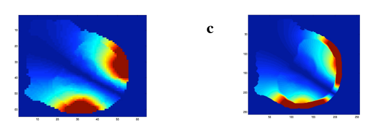


Potential (a) and current (b) distributions for 2 point current injection:
Resolution: 256x256x256 mesh:
Time: ~30 minutes
at PC (Pentium 3, 512 RAM,1.2 GHz)

MRI: FEMLAB RESULTS



FD ADI Algorithm: MRI - 3D Heterogeneous Case Study



2D cross-section of the 3D Poisson equation solution at 64x64x64 © and 256x256x256.) .
The effective conductivity in the air regions is 0.01% of the average head conductivity. Convergence: < 150 iterations.

SUMMARY

-FEMLAB is instrumental as a FE solver and/or a FE mesh generator for 3D phantoms and 2D realistic MRI geometries
-In case of 3D MRI data of the brain the inner structure must be simplified to be imported into FEMLAB
-Finite Difference ADI algorithm [4] has been identified as an appropriate choice for a fast solver in the forward problem

DISCUSSION AND FUTURE RESEARCH

When using FD methods one should be aware about the following "pros" and "cons":

-meshes are relatively easy to construct as the cubic/rectangular elements can be "mapped" directly from the voxels of the medical images (3D MRI scans);
-many anatomical details can be included as the computational load is based on the number of elements and not on the specifics of tissues differentiation.
- the "native" geometry for FD is rectangular, therefore the simplest way of implementing curved boundaries is to embed the complex object into a computational cubic domain. However, the redundant voxels can bring additional computational cost in terms of accuracy and speed.

Future work will involve refinement and parallel implementation of the FD ADI algorithm based solver for the forward problem with the aim of applications in the EEG/MEG inverse problem and other modalities (such as electrical impedance tomography and NIR diffusive optical tomography). It will include:

-investigation of FDM accuracy and rate of convergence against the bench-mark analytical models and FEM algorithms using FEMLAB and other FEM software;

- implementation of a parallel version of the current FORTRAN 77 code in C/C++ to run on computer cluster NEURONIC;

- implementation of a similar FD ADI algorithm for solving the photon migration equation the heterogeneous brain tissues, which is basic in NIRS modalities.

Reference

1.R.M. Gulrajani, Bioelectricity and Biomagnetism.New York: John Wiley & Sons, 1998
2.J. Jin, The Finite Element Method in Electromagnetics. New York: John Wiley & Sons, 1993
3.W.H Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, The Numerical Recipes in C: The art of Scientific Computing, 2nd edition. New York:Cambridge University Press, 1992
4. V.N.Abrashin, I.A.Dzuba, "Economical Iterative Methods for solving multi-dimensional problems in Mathematical Physics", Differential Equations, Vol. 30 , No.2, pp. 281-291.(translation from Russian).