Multi-Cluster, Mixed-Mode Computational Modeling of Human Head Conductivity

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Collaboration

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- Robert Frank

- Electrical Geodesic, Inc :

- Peter Lovely, Colin Davey, Pieter Poolman, Jeff Eriksen , and Don Tucker

Motivation

- Goal: To estimate the electrical conductivities of human head based on realistic segmented MRI or CT scans

Necessary for …

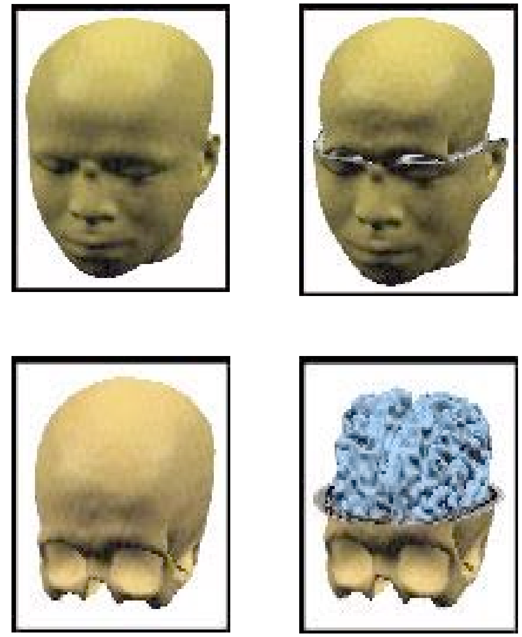
- Source Localization: find the electrical source generator for the potential that can be measured at the scalp
- Detecting abnormalities: cracks, holes, … etc

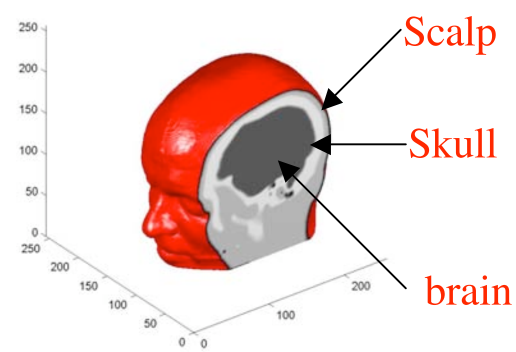
Building Computational Head Models

To relate the neural activity in the head to the EEG measurements on the scalp

- Three parts in constructing a human head model
  1. Geometry: Geometrical Model of the head with its tissue types
     - ➔ Sphere models: 4-sphere model, 3-sphere model
     - ➔ MRI or CT: determines the boundaries of the major head tissues
  2. Electrical Conductivity model: Assign a conductivity value for each tissue type
     - ➔ Homogenous: Assign an average value for the entire MRI segment
     - ➔ Known: For each tissue type it varies considerably
  3. Forward problem: Evolution of the potential within each tissue.

     Given the conductivities of the head tissue and the current sources, find the potential at each point in the head.





Computational Head Models: Forward problem

Continuous Solutions — Governing Equations, IC/BC

↓

Finite-Difference
Finite-Element
Boundary-Element
Finite-Volume
Spectral
— Discretization

↓

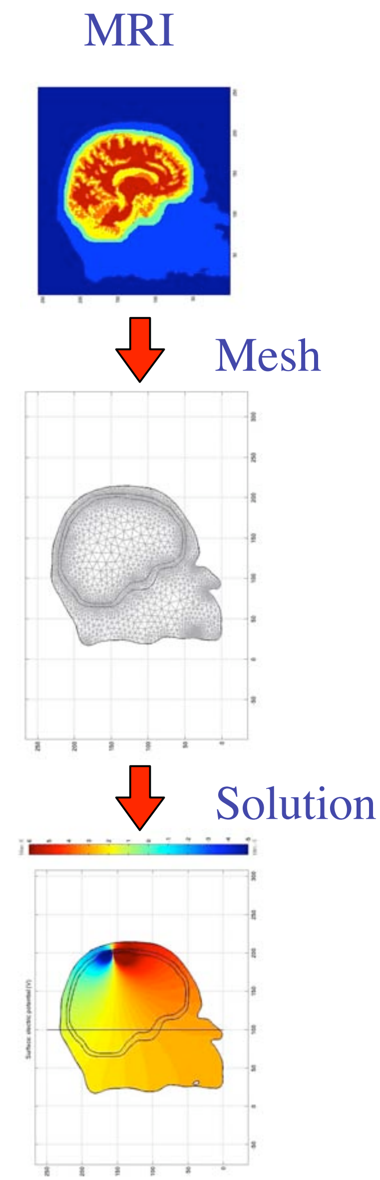
Discrete Nodal Values — System of Algebraic Equations

↓

Tridiagonal
Gauss-Seidel
Gaussian elimination
— Equation (Matrix) Solver

↓

ϕ (x,y,z,t)
J (x,y,z,t)
B (x,y,z,t)
— Approximate Solution



Computational Head Models: Forward problem

The governing equation is:

- The Poisson equation

$$\nabla \bullet (\sigma \nabla \phi) = \nabla \bullet J^s, \quad \text{in } \Omega$$

- With the boundary condition

$$\sigma(\nabla \phi) \bullet n = 0 \text{ , on } \Gamma_\Omega \ .$$

Where, $\sigma = \sigma_{ij}(x,y,z)$ is a tensor of the head tissues conductivity, J^s, current source.

Computational Head Models: Forward problem

Multi-component ADI Method:

- unconditionally stable in 3D
- accurate to $O(\tau + \Delta x^2 + \Delta y^2 + \Delta z^2)$

$$\frac{\phi_i^{n+1} - \overline{\phi^n}}{\tau} + \delta_x \phi_i^{n+1} + \delta_y \phi_j^n + \delta_z \phi_k^n = S$$

$$\frac{\phi_j^{n+1} - \overline{\phi^n}}{\tau} + \delta_x \phi_i^{n+1} + \delta_y \phi_j^{n+1} + \delta_z \phi_k^n = S$$

$$\frac{\phi_k^{n+1} - \overline{\phi^n}}{\tau} + \delta_x \phi_i^{n+1} + \delta_y \phi_j^{n+1} + \delta_z \phi_k^{n+1} = S$$

Here : $\overline{\phi^n} = (\phi_i^n + \phi_j^n + \phi_k^n)/3$

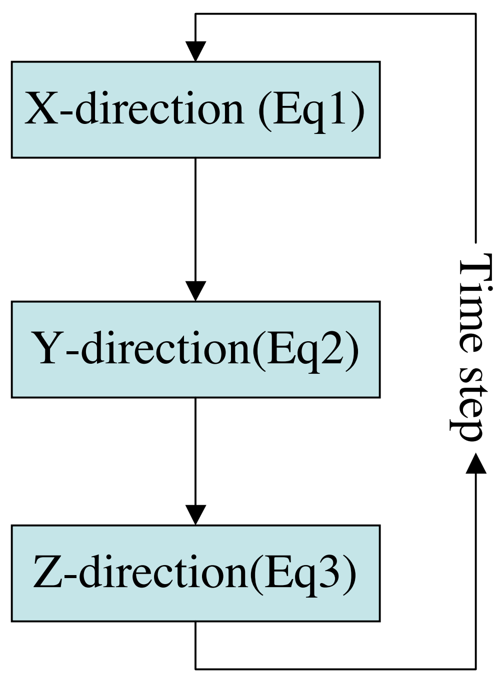
$\delta_{x,y,z}$ is notation for an 1D second order spatial difference operator

Reference: Abrashin et al, Differential Equations 37 (2001) 867

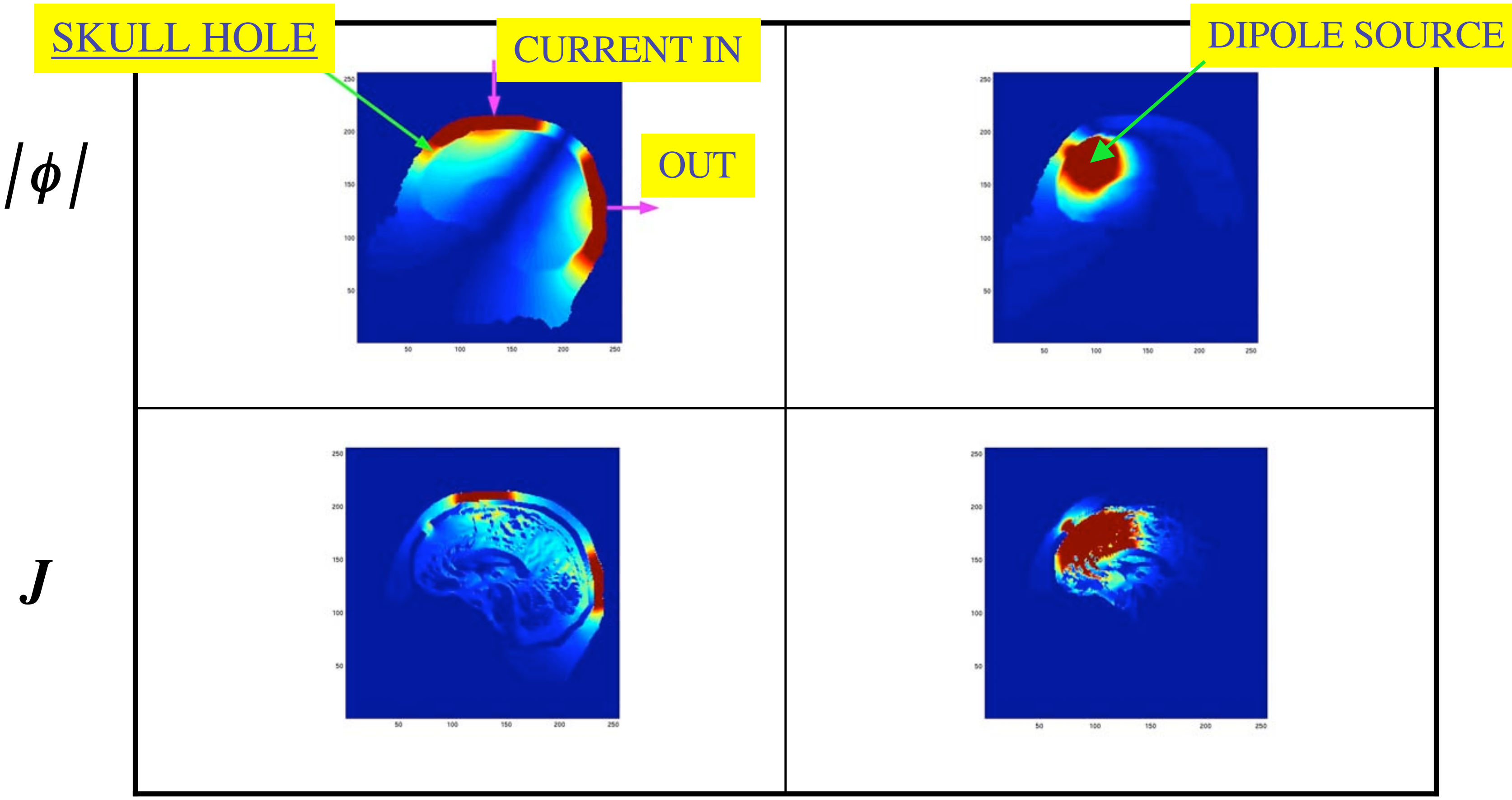
Computational Head Models: Forward problem

Multi-component ADI algorithm:

- Each time step is split into 3 substeps
- In each substep we solve a 1D tridiagonal systems



Computational Head Models: Forward problem: solution

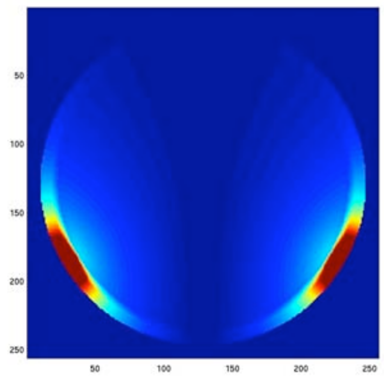


External Current Injection
(Electrical Impedance Tomography)

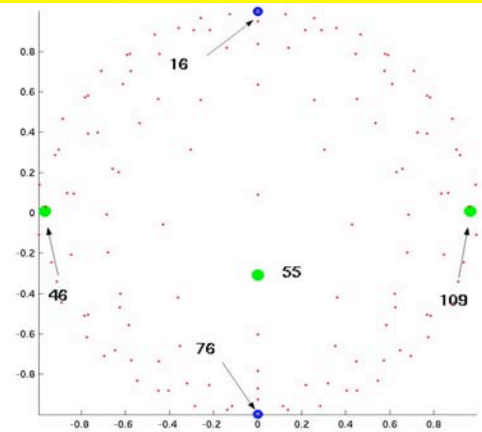
Intracranial Dipole Source Field
(Epileptic Source Localization)

Computational Head Models: Forward problem: Validation

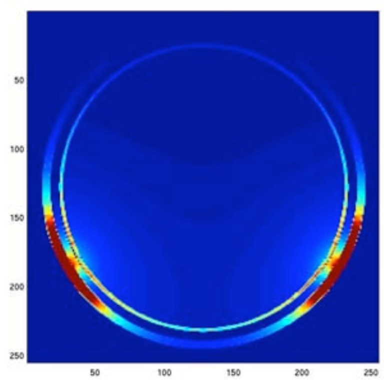
$|\phi|$



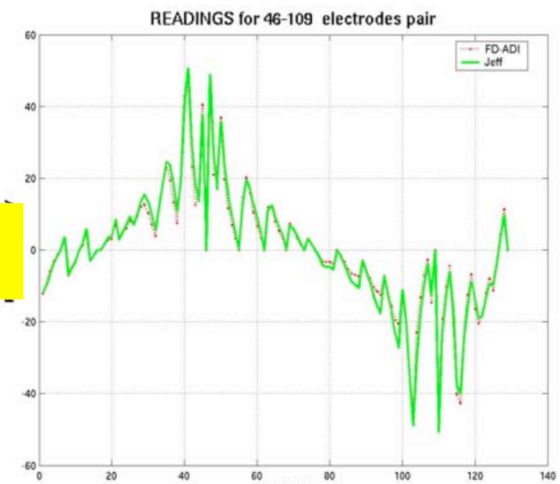
Electrode Montage: XY view



J



$|\phi|$



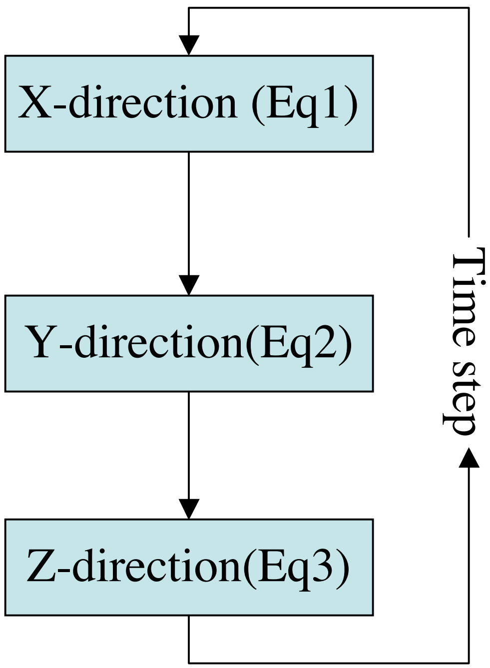
Electrode Number

Computational Head Models: Forward problem: Parallelization

- The computation to solve the system of equations in each substep is independent of each other
- Example: in the x direction we can solve the $N_y N_z$ equations concurrently on different processors
- The Parallel program structure is:

  For each time step
  - Solve $N_y N_z$ tridiagonal equations
  - Solve $N_x N_y$ tridiagonal equations
  - Solve $N_y N_z$ tridiagonal equations

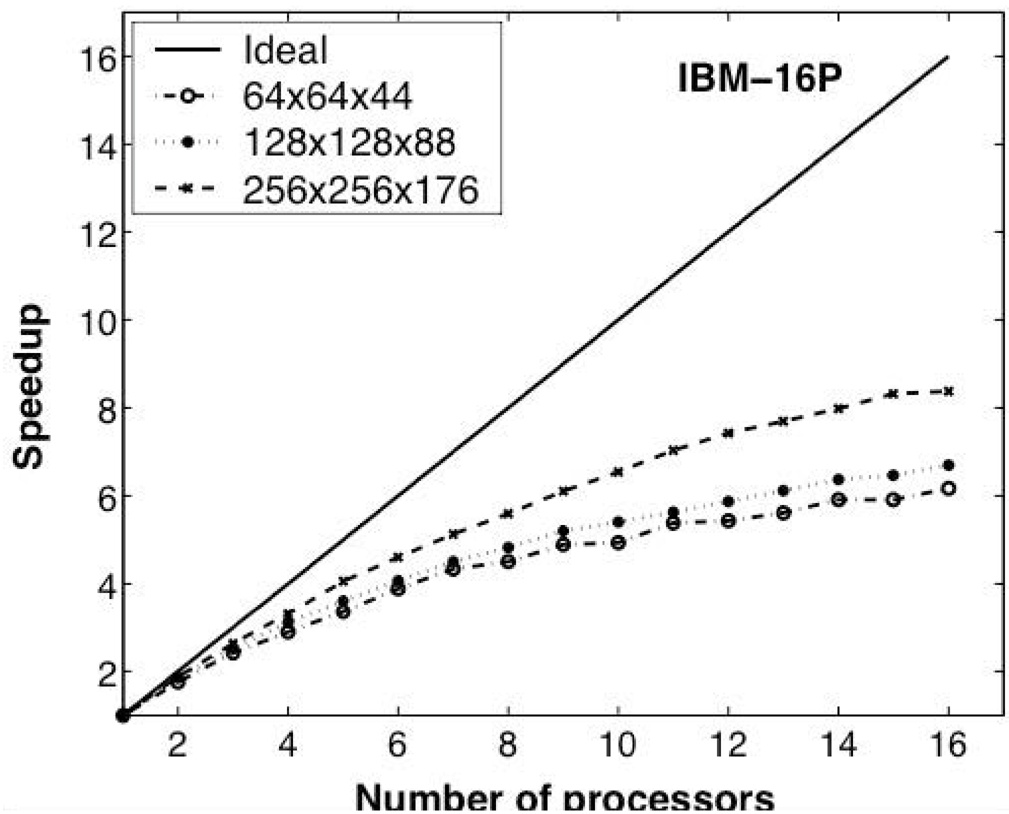
  End



- We used openMP to implement the parallel code in a shared memory clusters

Computational Head Models: Forward problem: Parallelization speedup

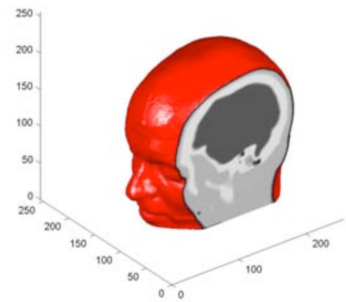
Forward Solution Speedup on IBM-P690



Computational Head Models: Inverse Problem

- Given the measured electric potential at the scalp V_i, the current sources and the head tissue geometry

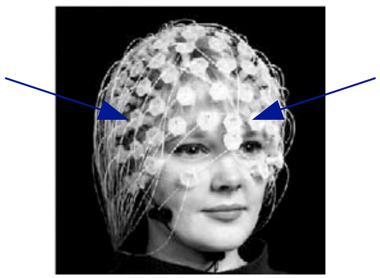
Estimate the conductivities of the head tissues



The procedure to estimate the tissue conductivities is:

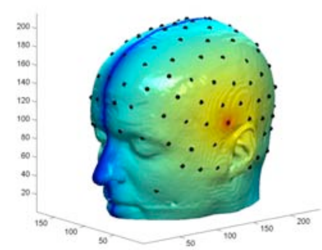
- Small currents are injected between electrode pairs
- Resulting potential measured at remaining electrodes
- Find the conductivities that produce the best fit to measurements by minimizing the cost function:

$$E = \left[\frac{1}{N} \sum_{i=1}^{N} (\phi_i^p - V_i)^2 \right]^{1/2}$$



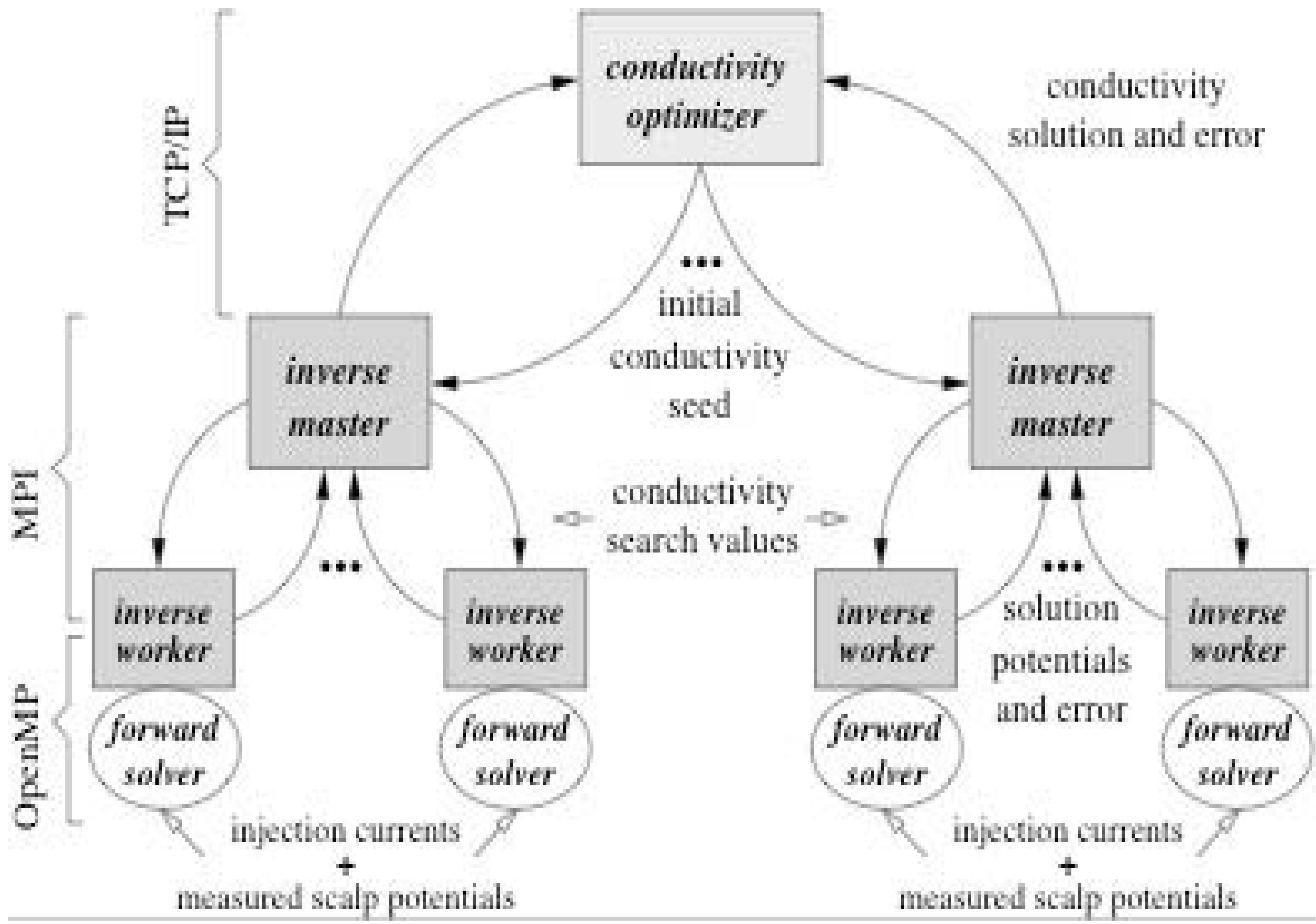
Measurements

- Computationally intensive



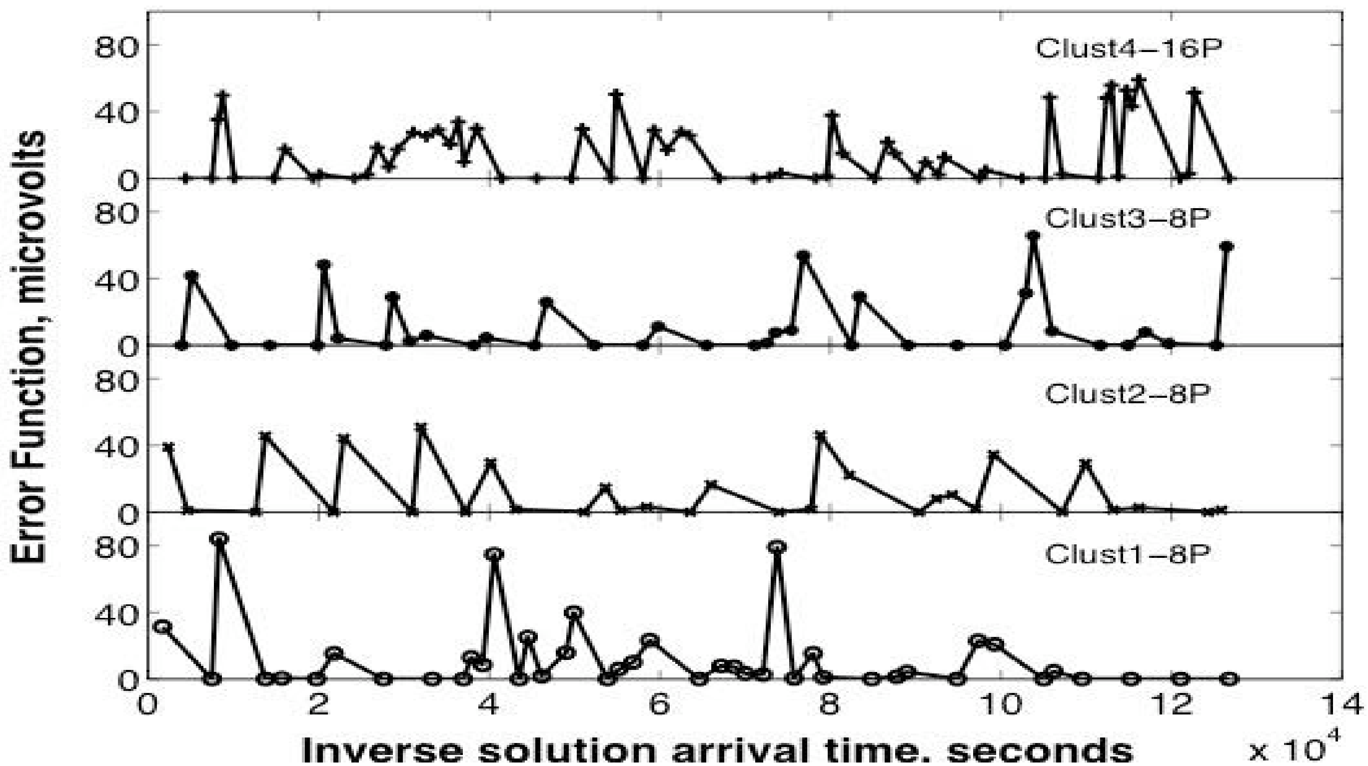
Computational model

Schematic view of the parallel computational system



Performance Statistics

Dynamics of Inverse Search



Performance Statistics

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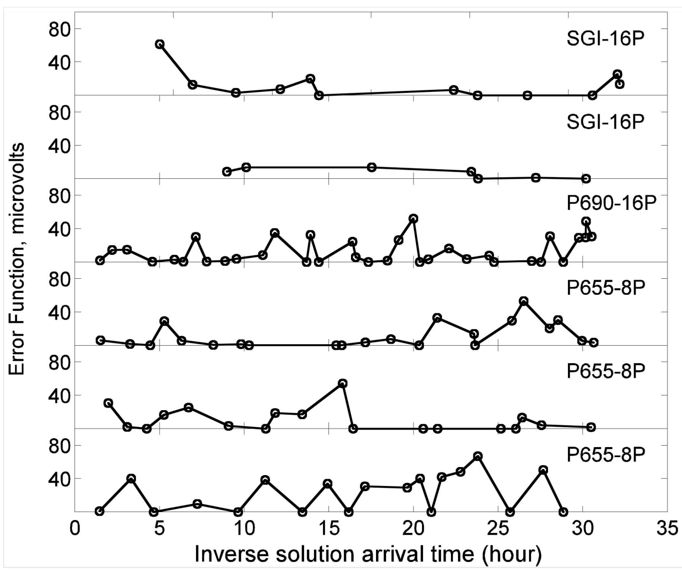
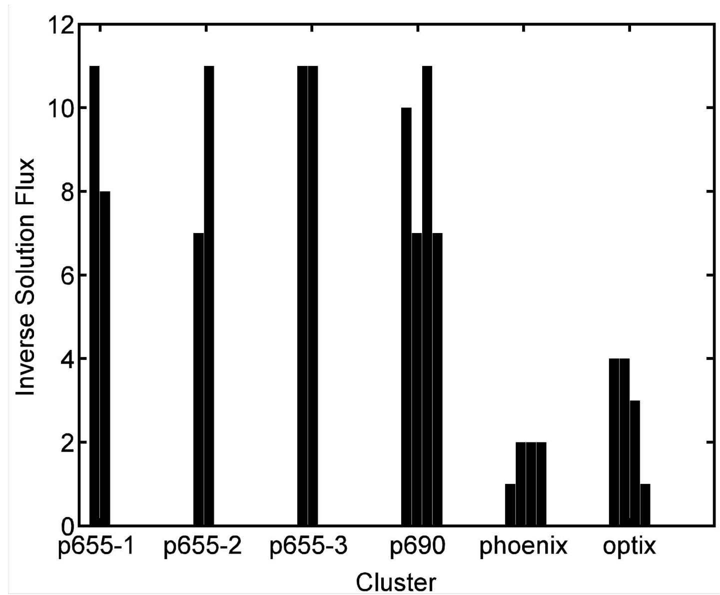
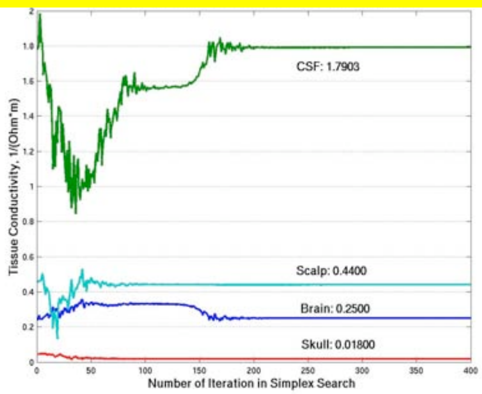




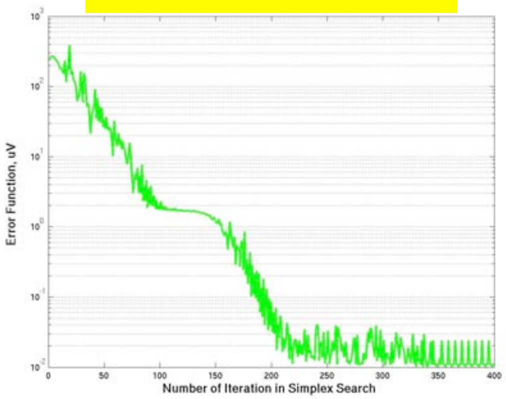
Inverse Problem: Simplex Algorithm

simulated data (real MRI)

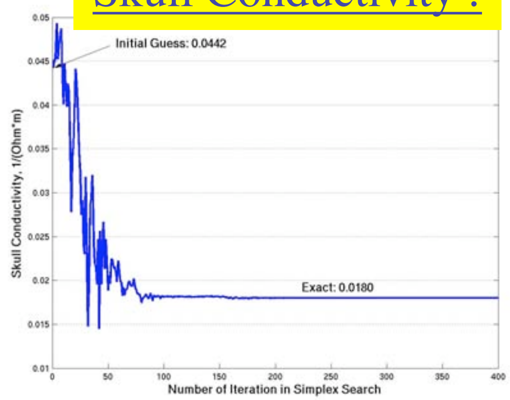
Dynamics of Inverse Solution:



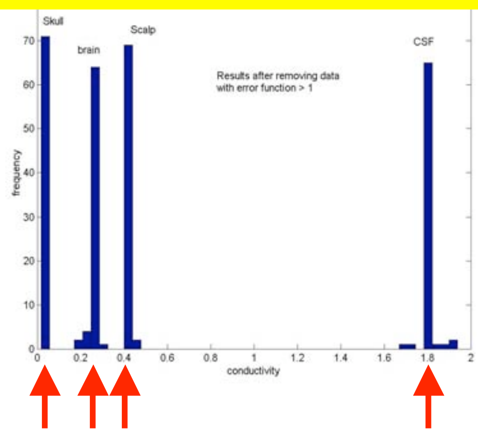
Error Dynamics:



Skull Conductivity :



Extracted Conductivities



Exact Values

Error Function to minimize:

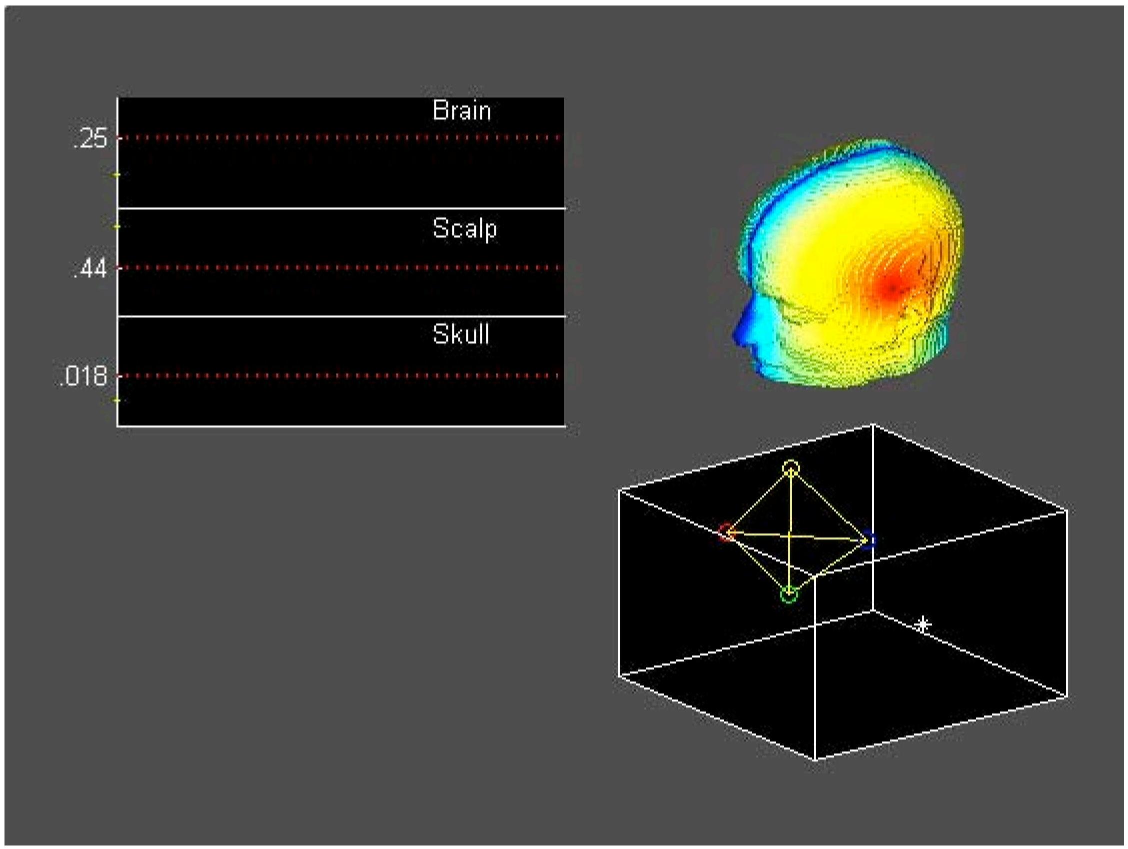
$$E = \left[\frac{1}{N} \sum_{i=1}^{N} (\phi_i^p - V_i)^2 \right]^{1/2}$$

Retrieved tissues conductivities

Tissue type	**σ(Ω⁻¹m⁻¹)**	**Δσ(Ω⁻¹m⁻¹)**
Brain	0.2491	0.0099
CSF	1.7933	0.0311
Skull	0.0180	0.00017
Scalp	0.4400	0.00024

Inverse Problem: Simplex Algorithm

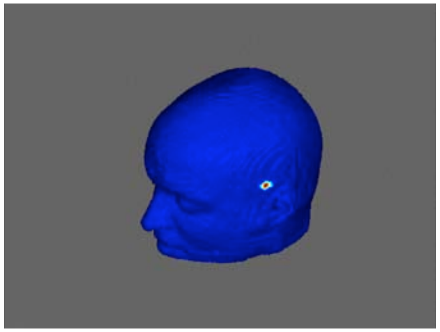
simulated data (real MRI)



Summary

- Finite Difference ADI algorithm based 3D solvers for the forward electrical have been developed and tested for variety of geometries;
- The electrical forward solver has been optimized and parallelized within OpenMP protocol of multi-threaded, shared memory parallelism to run on different clusters;
- The successful demonstrations of solving the nonlinear inverse problem with use of HPC for search and estimation of the unknown head tissues conductivity have been made for 4-tissues segmentation on the realistic MRI based geometry (128^3 resolution) of the human head;
- The work with experimental human data is in progress

Thank you ….



Questions